Faculty: Mr. M. K. Singh
S.S. College, Jehanabad

Electro Magnetic Theory

B.Sc (H) Physics Part II

Vector Analysis

1.1 Vector Algebra : The quantity having direction as well as magnitude ~~(length)~~ is called vector.

Eg: Displacement
Velocity
acceleration

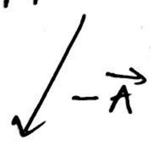
~~It is denot~~

* A vector is denoted by **A** (boldface)

or $\vec{A}$ (~~att~~ arrow above the letter)

* Magnitude of vector denoted by $|\vec{A}|$ or 'A'
  * In diagrams vectors are denoted by arrow: the length of the arrow is proportional to the magnitude of the vector, and the arrowhead indicates its direction.

* $-\vec{A}$ (Minus $\vec{A}$) is a vector with the same magnitude as $\vec{A}$ but of opposite direction as shown below



* Note that vectors have magnitude and direction but not location: Therefore on a diagram we can slide the arrow around at will, as long as we donot change its length or direction.

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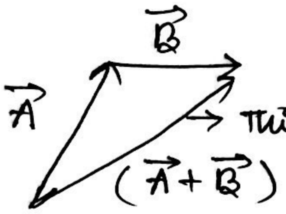
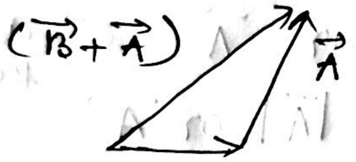
→ Now we define vector operations

(i) Addition of two vectors :

Place the tail of $\vec{B}$ at the head of $\vec{A}$; the sum $\vec{A}+\vec{B}$ is the vector from the tail of $\vec{A}$ to the head of $\vec{B}$ as shown below



This vector represents sum of $\vec{A}$ and $\vec{B}$



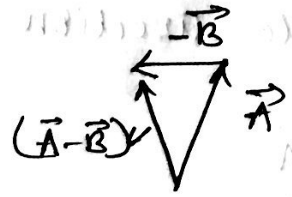
Rules for addition of two vectors

(i) Commutative : $\vec{A}+\vec{B} = \vec{B}+\vec{A}$

(ii) Associative : $(\vec{A}+\vec{B})+\vec{C} = \vec{A}+(\vec{B}+\vec{C})$

Subtraction of two vector

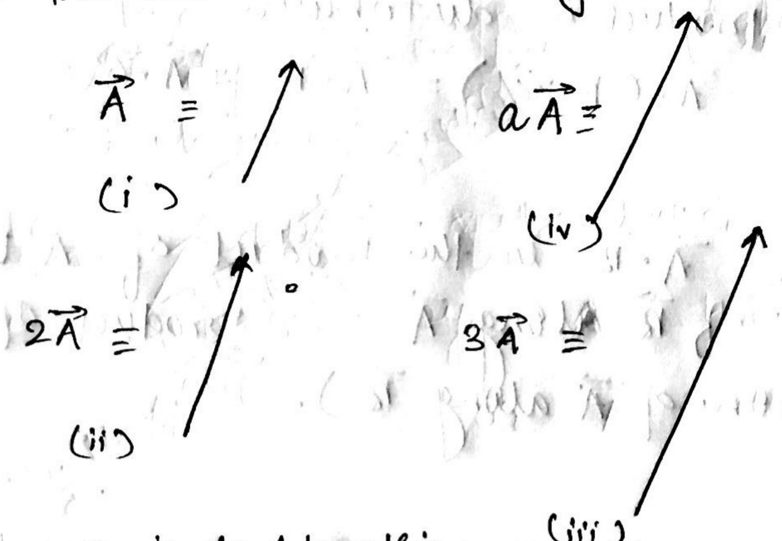
~~Add~~ $\vec{A}-\vec{B}$



It is basically addition of two vectors by changing the direction one of them.

(ii) Multiplication by a scalar.

Multiplication of a vector by a scalar: Multiplication of a vector by a positive scalar 'a' multiplies the magnitude but leaves the direction unchanged (as shown in figure below)



We see that magnitude of length in fig(i) the smaller compared to fig(ii) and fig (iii)

* The scalar multiplication is distributive

$$a(\vec{A}+\vec{B}) = a\vec{A} + a\vec{B}$$

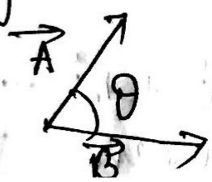
(iii) Dot product of two vectors

The dot product of two vectors is defined as

$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$$

* $|\vec{A}|$ is magnitude of $\vec{A}$
* $|\vec{B}|$ is magnitude of $\vec{B}$

and θ is the angle they form when placed tail to tail as



* Note that $\vec{A}\cdot\vec{B}$ is itself a scalar (hence the alternative name scalar product).

* The dot product is commutative

$$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$$

* The dot product is distributive

$$\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$$

Geometrically, $\vec{A}\cdot\vec{B}$ is the product of A times the projection of $\vec{B}$ along $\vec{A}$ (or product of B times the projection of $\vec{A}$ along $\vec{B}$).

* When $\theta = 0^\circ$ the two vectors are parallel

then $\vec{A}\cdot\vec{B} = |\vec{A}|\,|\vec{B}|\cos\theta$

$$= AB\cos 0^\circ = AB$$

* When $\theta = 90^\circ$ the two vectors are perpendicular

then $\vec{A}\cdot\vec{B} = |\vec{A}|\,|\vec{B}|\cos 90^\circ$

$$= 0$$

$$\boxed{\because \cos 90^\circ = 0}$$

<u>Example</u> Let $\vec{C} = \vec{A}-\vec{B}$ find $\vec{C}\cdot\vec{C}$

$$\vec{C}\cdot\vec{C} = (\vec{A}-\vec{B})\cdot(\vec{A}-\vec{B})$$

$$= \vec{A}\cdot\vec{A} - \vec{A}\cdot\vec{B} - \vec{B}\cdot\vec{A} + \vec{B}\cdot\vec{B}$$

[By associative law of multiplication]

$$= A^2 + B^2 - 2AB\cos\theta$$

* θ is angle between $\vec{A}$ and $\vec{B}$

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